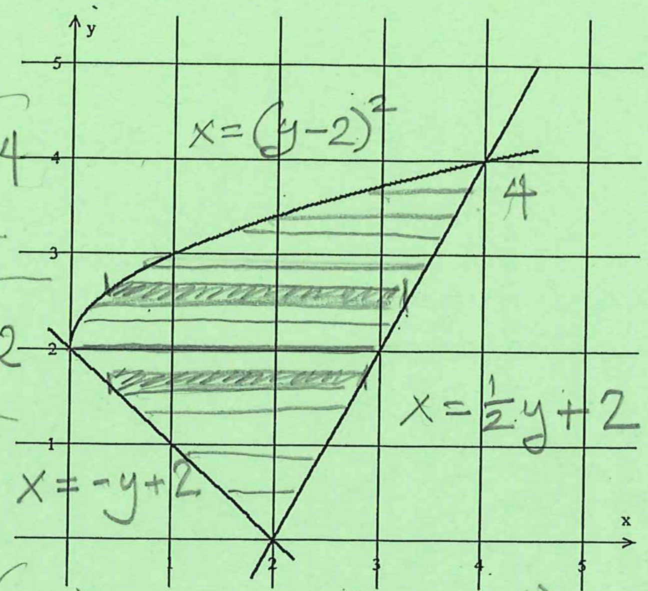
Math 265B: Quiz 1 **Name:** KEY

1. (6 pts) Consider the region bounded by $y=\sqrt{x}+2$, $x=-y+2$, and $x=\frac{1}{2}y+2$, as shown.

Set up the integral(s) <u>with respect to **y**</u> that give the area of this region. Clearly show the slices. Evaluate the integrals. (You may find the intersection points "by inspection".)



$y=4$

$y=2$

$$A=\int_0^2 \tfrac{1}{2}y+2-(-y+2)\,dy$$

$$+\int_2^4 \tfrac{1}{2}y+2-(y-2)^2\,dy \quad (*)$$

$$=\int_0^2 \tfrac{3}{2}y\,dy+\int_2^4 -y^2+\tfrac{9}{2}y-2\,dy$$

$$=\tfrac{3}{4}y^2\Big|_0^2+\left[-\tfrac{1}{3}y^3+\tfrac{9}{4}y^2-2y\right]_2^4$$

$$(*)\ \tfrac{1}{2}y+2-(y^2-4y+4) = -y^2+\tfrac{9}{2}y-2$$

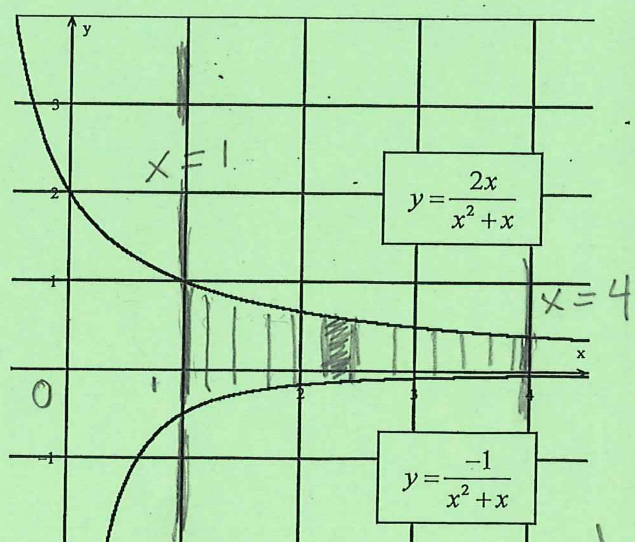
$$=\tfrac{3}{4}(2^2-0^2)-\tfrac{1}{3}(4^3-2^3)+\tfrac{9}{4}(4^2-2^2)-2(4-2)$$

$$=3-\tfrac{56}{3}+27-4=26-\tfrac{56}{3}=\tfrac{78}{3}-\tfrac{56}{3}=\boxed{\tfrac{22}{3}}=\boxed{7\tfrac{1}{3}\text{ unit}^2}$$

(reasonable!)

2. (4 pts) Set up and evaluate the integral to find the exact area of the region bounded by $y=\frac{2x}{x^2+x}$, $y=\frac{-1}{x^2+x}$, x = 1, x = 4

Clearly show the slices.



$$A=\int_1^4 \frac{2x}{x^2+x}-\frac{(-1)}{x^2+x}\,dx$$

$$=\int_1^4 \frac{2x+1}{x^2+x}\,dx$$

$u=x^2+x$

$du=2x+1$

$x=1$, $u=2$

$x=4$, $u=20$

$$=\int_2^{20}\frac{1}{u}\,du$$

$$=\ln|u|\Big|_2^{20}=\ln(20)-\ln(2)=\boxed{\ln(10)}\approx 2.3\text{ unit}^2$$

reasonable!