Math 265B: Quiz 2 (Volumes) Name: KEY

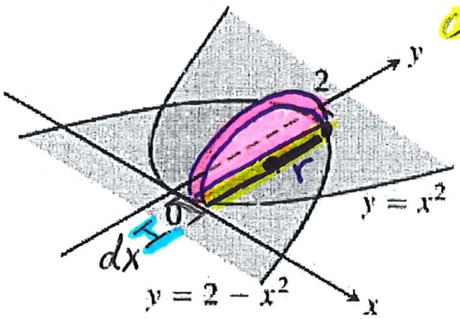
1. (4 pts) Set up and evaluate the integral needed to find the volume of the solid illustrated below:
   - The base is the region bounded by $y = x^2$ and $y = 2 - x^2$
   - The cross-sections are half-circles, diameter parallel to the y-axis, as shown.

half-circle: $A = \pi r^2/2$

$d = 2 - x^2 - x^2$
$= 2 - 2x^2$

$r = \frac{d}{2} = \frac{2-2x^2}{2}$

$r = 1 - x^2$



Intersection points: $2 - x^2 = x^2$
$2 = 2x^2$
$x^2 = 1 \Rightarrow x = \pm 1$

$V = \int \text{area} \cdot \text{thickness}$

$= \int_{-1}^{1} \frac{\pi r^2}{2} \cdot dx$

$= \int_{-1}^{1} \frac{1}{2}\pi (1 - x^2)^2 \, dx$

$= 2\left(\frac{1}{2}\pi\right) \int_0^1 1 - 2x^2 + x^4 \, dx$

$= \pi \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5 \right]_0^1 = \boxed{\frac{8\pi}{15}} \ \boxed{\text{unit}^3}$

2. (6 pts) Consider the following region bounded by $y = 4x$ and $y = x^3$ (x > 0), as shown.

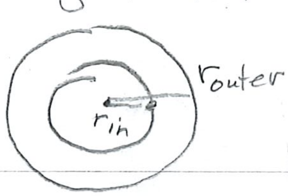
<u>Sketch</u> (on the first figure) the solid of revolution formed when the region is rotated about the the y-axis. Then, set up each integral for finding the volume using the stated method. You do not have to evaluate the integrals.

(a) The Disk/Washer Method:

$V = \int_0^8 \pi \left[y^{1/3} \right]^2 - \pi \left[\frac{y}{4} \right]^2 dy$

$\boxed{= \pi \int_0^8 y^{2/3} - \frac{1}{16} y^2 \, dy}$

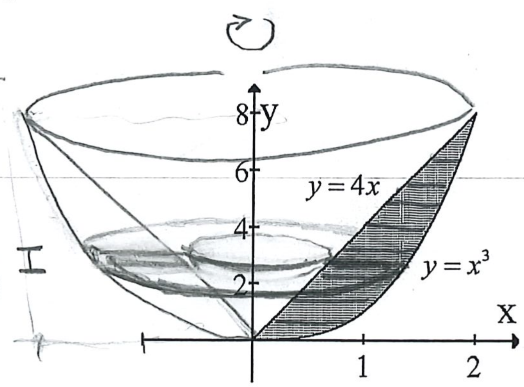
Cross-section of Washer:



$r_{inner} = \frac{y}{4}$

$r_{outer} = \sqrt[3]{y} = y^{1/3}$

dy



Intersection: $4x = x^3$
$x^3 - 4x = 0$
$x(x^2 - 4) = 0$
$x = 0, \ x = \pm 2$
$y = 0, \ y = \pm 8$

(b) The Shell Method:

$V = \int 2\pi r h \cdot \text{thickness}$

$= 2\pi \int_0^2 x(4x - x^3) \, dx$

$\boxed{= 2\pi \int_0^2 4x^2 - x^4 \, dx}$