Math 265B: Review for Test 1 (Spring, 2020)

Calculator: Only *non-graphing, scientific calculators* may be used on the exam.

Concepts on Exam:

1. u-Substitution/Reversing the Chain Rule (Section 5.5): Use Substitution to find anti-derivatives/evaluate definite integrals. Know how to check your work (you'll be asked to check at least one of the anti-derivatives).

2. Area of a region (Section 6.2): Find the area between curves. Sketch the region then slice it up

vertically (thickness of slices is Δx), $Area = \int_{x = \text{left}}^{x = \text{right}} \left[f_{upper}(x) - g_{lower}(x) \right] dx$

horizontally (thickness of slices is Δy $Area = \int_{y = \text{lower}}^{y = \text{upper}} \left[f_{right}(\text{y}) - g_{left}(\text{y}) \right] dy$

Recognize when the boundaries have a point where they change and thus a second integral is needed to give the area; set up the integrals for such an area.

3. Volumes by slicing (Section 6.3): Be able to find the volume of a solid created by arranging shapes with a particular cross-sectional shape with a cross-sectional *area* (such as a square or triangle) on a base region in the xy-plane.

. Total Volume = $\int_{\text{beginning slice}}^{\text{ending slice}} \text{Area of slice} \cdot \text{thickness}$ (thickness = dx or dy)

4. Solids of Revolution (Sections 6.3 and 6.4):
(a) Be able to *sketch* a solid of revolution, including showing a typical slice and what shape it assumes (disk, washer, or cylinder) when it is rotated about an axis.
(b) Be able to find the volume, V, of a solid of revolution by both the Method of Disks/Washers and the Method of Shells.

$V_{disks/washers} = \int_{\text{slices begin}}^{\text{slices end}} \left[\pi r_{outer}^2 - \pi r_{inner}^2 \right] thickness$ (thickness = dx or dy, depending on how you slice it)

$V_{shells} = \int_{\text{slices begin}}^{\text{slices end}} 2\pi \cdot r \cdot h \cdot thickness$, where r = the radius of the shell and h = the height of the shell.

Note: thickness = dx only, on the exam

5. (**Arc)Length of a curve (Section 6.5):** Be able to derive or explain parts of the derivation of the arclength formula (as was shown in class). Apply the formula in setting up the integral to find the length of a particular curve on a given interval.

$$Arclength = \int_a^b \sqrt{1 + \left[f'(x) \right]^2} \; dx$$

6. Physical Applications of Integration

- **Mass/ Density problems:** If you have a "thin rod" made out of some substance that has $\rho(x)$ = linear density = mass per unit length (kg/m, for example), find the total mass.

$$\text{Total mass} = \int_{\text{beginning slice}}^{\text{ending slice}} \rho(x)\cdot dx$$

- **Work problems:**
  - Spring problems: Know Hooke's Law: $F(x) = kx$
    Be able to find the spring constant, k
    Be able to find the total work done in compressing or expanding the spring,

$$\text{Work} = \int_{\text{start position}}^{\text{stop position}} F(x)\cdot dx$$

  - Lifting problems (emptying a tank). Be sure to do a ***sketch*** to reason these problems out!

$$\text{Work} = \int_{\text{location of first slice}}^{\text{location of last slice}} (\text{weight of slice})\cdot(\text{distance slice must be moved})$$

$$= \int_{y\,=\,\text{bottom slice}}^{y\,=\,\text{top slice}} \rho g\cdot A(y)\; D(y)dy$$

where $\rho =$ the density of the liquid
g = acceleration of gravity
$D(y)$ = distance the slice must move
$A(y)$ = the face area of slice (looking down on it)

The hydrostatic force topic will be featured as a take home problem on the exam.

- **Hydrostatic force problems:** Given a dam or a submerged object, the pressure, p, is given by the formula $p = \rho gh$, where $\rho =$ the density of the liquid, g = acceleration of gravity, and h = the depth below the surface.

  On a ***sketch*** of the dam or the submerged object, slice it horizontally then focus on one slice which is at a fixed depth (where the pressure is constant).

$$\text{Force} = \int_{\text{depth of first slice}}^{\text{depth of last slice}} (\text{pressure})\cdot(\text{area of slice})$$

$$\int_{y\,=\,\text{bottom slice}}^{y\,=\,\text{top slice}} \rho g\cdot h\cdot w(y)\; dy$$

where h = depth <u>in terms of y</u>

w(y) = width of the slice