Math 256B: Review for Test 2
Note: Only scientific calculators are allowed on the exam.

Integration Techniques and Strategy…methods we've studied:

The exam will have integration problems where the method to use is specified <u>and</u> problems where you will have to determine the best method to use. (Look up the old exams on my website to see see the possible format of the questions).

Caution: Numerical methods aren't covered on the old (Test 2) exams and Improper Integrals are. For our test on Tuesday, 3/10/20, there will be Numerical Methods questions on the In-Class exam and Improper Integral questions on the Take Home.

Manipulation of Fractions:

- Use Long Division for Improper Fractions
- Break up a two-or-more-term numerator and integrate each term
- You should know the integration formulas for inverse sine, inverse tangent, and inverse secant, in addition to the other Basic Integral Formulas we've used up to this point.

- **u-Substitution (or Reversing the Chain Rule):** Always look for this first! Train your eye to see a function, u, and its derivative, du, in the integrand. Try NOT to use mechanical means of doing u substitution (i.e., DON'T solve for dx and substitute that into the integral).

Integration by Parts: Memorize the Formula $\int u \cdot dv = uv - \int v \cdot du$

Which function should be u? use LIATE

If there's a solo function in the integrand (e.g. $\int \ln(x)\, dx$ or $\int \tan^{-1}(x)\, dx$) then let u = the solo function and dv = 1.

There will be problems where you have to use the formula directly and others where it would be much easier to use a table (the Column Method).

Partial Fraction Decomposition (PFD)

You should know the <u>decomposition formats</u> for the <u>single</u> and <u>repeated</u> linear factors and for one irreducible quadratic factor (not repeated) and be able to set up and solve and solve for the A,B,C, etc. values to decompose a rational function integrand.

Trigonometric Identities

Memorize the Pythagorean Identities (remember they all derive from $\cos^2 x + \sin^2 x = 1$)

What will be given: Half-Angle Identities for sine and cosine and Double-Angle Identities for sine and cosine

- If **<u>all</u>** the powers on sine and cosine are **<u>even</u>**, then apply Half-Angle Identities to reduce the powers.
- For **<u>odd powers</u>** or **<u>mixture of even and odd powers</u>**, "peel off" a *du* from the odd-powered function, then use Pythagorean Identities to get a *u, du* set-up.

Trig Substitution

Goal: Use a substitution to convert a binomial into a Pythagorean Trig ID, then collapse the binomial into a single term. This should result in a "simpler" integrate (at least one that's integrable).

Integrate the trig-based integrand then <u>*go back to the x's*</u> by sketching a triangle with the sides based on the original substitution.

Numerical Methods (for approximating the value of an integral)

Be able to approximate the value $\int_a^b f(x)dx$ using any of the following methods (summation formulas will be given):

Left Sum,
Right Sum,
MidPoint,
Trapezoid,
Simpson's Rule.

Be able to expand the summation (as shown in the notes) for a given n-value (number of subintervals).

Show work to find Δx (Memorize: $\Delta x = \dfrac{b-a}{n}$

Know the geometry of each method; i.e., be able to sketch the rectangles, trapezoids, or parabolic curves on a given graph (see notes).

Be able to find the Error for the approximation.

Know the factor that the error decreases by for each method (see notes).

Improper Integrals (this will be on the <u>Take Home</u> part of the exam):

There are two types of improper integrals:

- **Infinite intervals**: Integrals have **infinity** as an "endpoint" of integration (the interval you're integrating over is infinite)
- **Unbounded integrand:** There is a value in the interval that causes division by zero (a "singularity") in the integrand (the integrand function "blows up", i.e., it goes to infinity at this "singularity")

  The undefined value in the integrand can either be an endpoint in the interval or <u>somewhere</u> within the limits of integration.

To determine convergence/ divergence **by evaluating** the integral:

1. Rewrite the integral as a limit with b → infinity or b → singularity,
2. Evaluate the integral
3. T<u>ake the limit</u>. Note if the "singularity" , x = b, is in <u>between</u> the limits of integration you'll need to do a 2-integral analysis,

i.e. $\lim_{x \to b^-} \int_a^b f(x)dx$ and $\lim_{x \to b^+} \int_b^c f(x)dx$

DON'T DROP THE LIMIT NOTATION IN THE MIDDLE OF THE PROBLEM!