Math 265B: Review for Test 3

Summary of Concepts (Chapters 9 [10] and 10 [11]

Sequences:

- Given a sequence, find the nth term

- Find the limit of a sequence.

    - There are two approaches to finding limits as n goes to infinity. One is to find the limit intuitively "by inspection", using the concept of dominance, plugging in a large number, etc. . These are a loose way to find limits but are very useful.

    - The other approach is to find the limit formally using analytic methods such as those mentioned above. On the exam, I will ask you to find limits "formally" in some problems. Later, when you're applying the Tests for Convergence, you may (and should be able to!) find limits by inspection.

        - Use analytic (formal) methods to find the limit of a sequence. You will have to be able to apply techniques such as algebraic manipulation, the Squeeze Theorem, and L'Hopital's Rule.

        - Be able to identify and explain why the limit of a sequence doesn't exist; specifically, note when the terms grow without bound, or oscillate with no damping, or oscillate without bound and thus don't approach a single value as n increases.

        - Know the Ranking Theorem and apply it in finding limits.

Infinite Series: (these are series of constants)

- Given an expanded series, be able to find the kth term of the series, and put it into $\sum a_k$ notation.

- Be able to find partial sums, S_n, of a given infinite series. Use the partial sum to determine convergence/divergence of the series. Examples: Telescoping Series, Alternating Series.

- Be able to determine and PROVE the convergence or divergence of a series using the following tests:
    - Geometric Series Test…if the Geo. Series converges, be able to find the sum as well.
    - Integral Test
    - p-Test
    - Direct Comparison
    - Limit Comparison
    - Ratio Test
    - Alternating Series Test

- Be able to explain, intuitively, what each of the tests above tells you about the behavior of the series and why the series does or does not converge based on that behavior.

- Be able to determine whether an alternating series converges absolutely, converges conditionally or diverges.

- Know that the "tails" of series are what determine divergence or convergence of a series

Power Series: (these are series that contain a variable…looks like a polynomial that has infinitely many terms)

- Given a power series, determine (a) the radius of convergence and (b) the interval of convergence

- Explain what the interval of convergence means.

- Find the derivative of a series

- Integrate a series

Taylor (MacLaurin) Polynomials and Taylor (MacLaurin) Series:

- Given a function *f*, find the Taylor Polynomial or Series, expanded about either $x = 0$ (These Taylor series are also called "MacLaurin Series" or $x = a$ *(These are general Taylor Series and are not called MacLaurin Series)*

- Given the graph of a function, identify the value or the sign of the coefficients of the Taylor Polynomial/Series for specified terms:

  Specifically, if the Taylor series is expanded about $x = 0$ (MacLaurin series), then
  - the function value and the Taylor Polynomial value must be the same at the point of expansion
    - P(0) = f(0)
  - the slope of the function at $x = 0$ is the coefficient of the linear term in the Taylor Polynomial/Series.
  - the concavity of the function at $x = 0$ tells* us about the coefficient of the quadratic term in the Taylor Polynomial/Series ((

- Memorize the Taylor series (including the nth terms) for e^x, cos(x), sin(x)

- Be able to find the Taylor series for a new function by manipulating a known series.