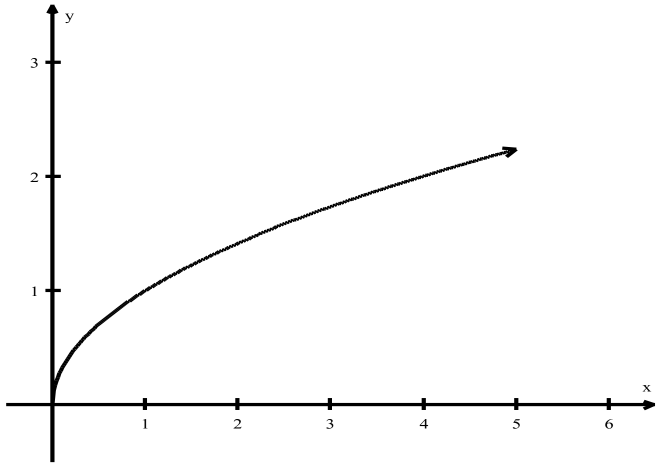
Math 265B: Numerical Methods of Integration (Section 7.7)

Using the Left Sum and Right Sum Rules, find an approximation for $\int_1^4 \sqrt{x}\, dx$ and the error

Use n = 1, 3 by hand (illustrate these), then n = 6, 60, 600 using Wolfram Alpha.

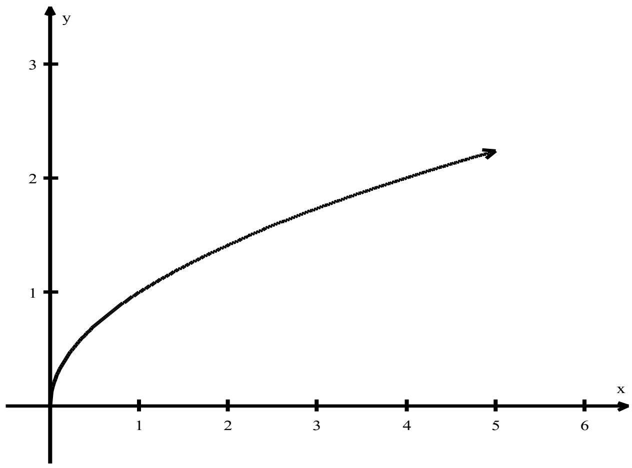
Left Sum:

$$\int_a^b f(x)\, dx \approx Left(n) = \sum_{k=0}^{n-1} f(x_k)\Delta x$$



Right Sum:

$$\int_a^b f(x)\, dx \approx Right(n) = \sum_{k=1}^{n} f(x_k)\Delta x$$



Error = Estimated value – True Value

n	**Left(n)**	**Right(n)**	**E(n) (Left)**	**E(n) (Right)**
1				
3				
6				
60				
600				

How does the error change as the number of subdivisions increases?

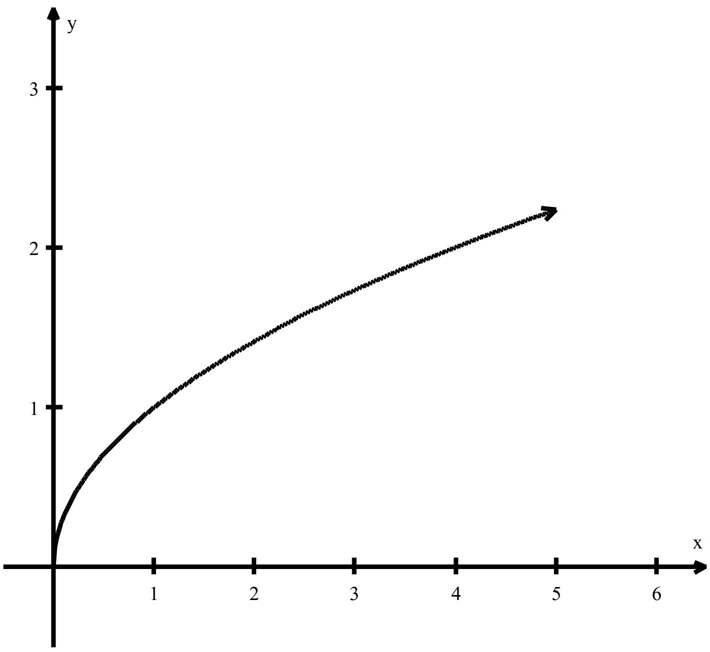
Based on the error, what could you do with the Left and Right Sums to create a new rule and thus reduce the error significantly?

Trapezoid Rule:

$$\int_a^b f(x)\,dx \approx Trap(n) = \left[\tfrac{1}{2} f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \tfrac{1}{2} f(x_0) \right] \Delta x \qquad \text{Alternately,} \qquad Trap(n) = \frac{Left(n) + Right(n)}{2}$$

Find the approximation for $\int_1^4 \sqrt{x}\,dx$ Use the Trapezoid Formula (not the averaging technique).

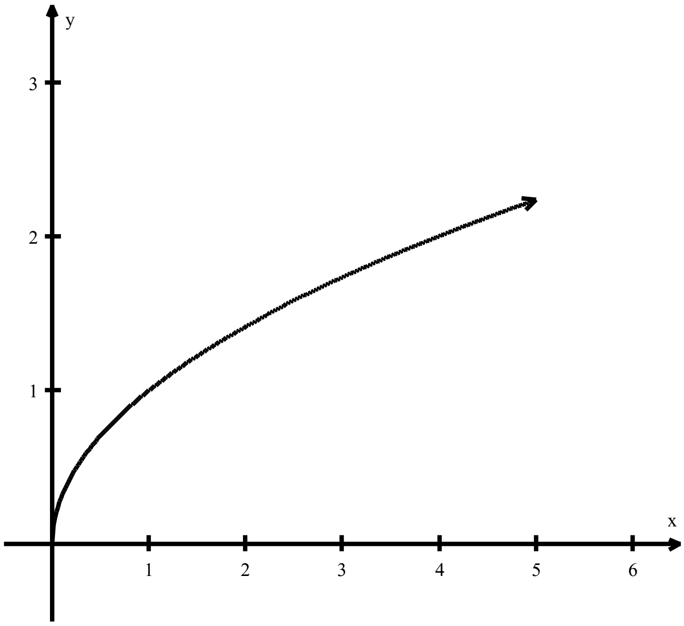
Use n = 1, n = 3 by hand (illustrate these), then n = 6, 60, 600 using Wolfram Alpha.
Fill in the table on the next page.



Midpoint Rule:

$$\int_a^b f(x)\,dx \approx Mid(n) = \sum_{k=1}^{n} f\left(\frac{x_{k-1}+x_k}{2}\right)\Delta x\,.$$

Find the approximation for $\int_1^4 \sqrt{x}\,dx$ Use n = 1, n = 3 by hand (illustrate these), then n = 6, 60, 600 using Wolfram Alpha. Fill in the table on the next page.



n	Trap(n)	Mid(n)	E(n) (Trap)	E(n) (Mid)
1				
3				
6				
60				
600				

How does the error change as the number of subdivisions increases?

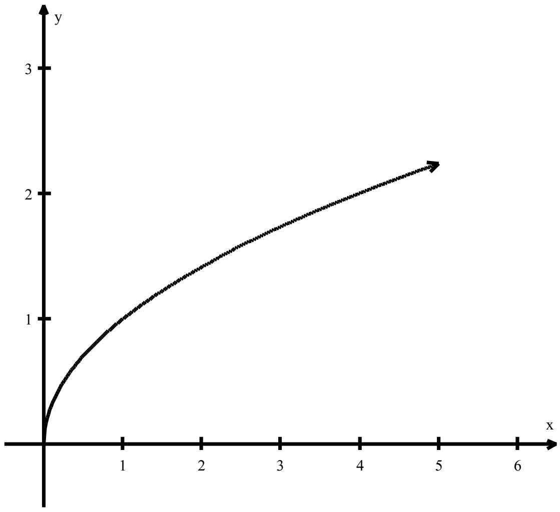
Based on the error, what could you do with the Trapezoid and Midpoint Sums to create a new rule and thus reduce the error significantly?

Simpson's Rule: *n must be an **even** integer* to apply Simpson's Rule**.**

$$\int_a^b f(x)\,dx \approx Simp(n) = \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n) \right] \frac{\Delta x}{3}$$

Alternately, $Simp(n) = \dfrac{2Mid(n) + Trap(n)}{3}$ or $S(2n) = \dfrac{4T(2n) - T(n)}{3}$ (book's method - meh)

Find the approximation for $\int_1^4 \sqrt{x}\,dx$ Use n = 2, 6, (illustrate), then n = 60, 600 using Wolfram Alpha.



n	Simp(n)	E(n)
2		
6		
60		
600		

How does the error change as the number of subdivisions increases?