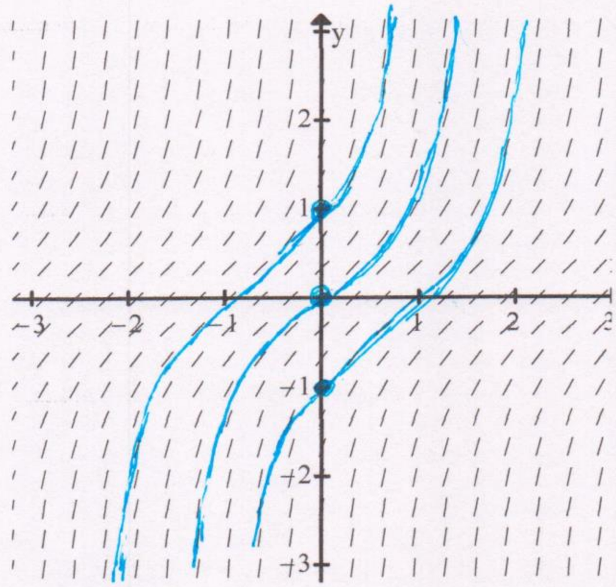
Math 265B: Differential Equations, Slope Fields and Analytical Solutions

Consider the following four differential equations:

(i) $y'(t) = \dfrac{t}{2+y}$ (ii) $y'(t) = \cos(t+y)$ (iii) $y'(t) = 1 + y^2$ (iv) $y'(t) = ty$

(a) Match each differential equation with the corresponding slope field. (See next page for two more slope fields)
(b) For each differential equation, sketch the solution curves that pass through each of the points (0, 0), (0, -1) and (0, 1).
(c) Find the General Solution for each differential equation using Separation of Variables, **if possible.**

1.



(iii) Matches (iii) $y' = 1 + y^2$

Note that $y' \neq 0$ anywhere on this slope field which matches the fact that $1 + y^2 \neq 0$ always

Solution: $\dfrac{dy}{dt} = 1 + y^2$

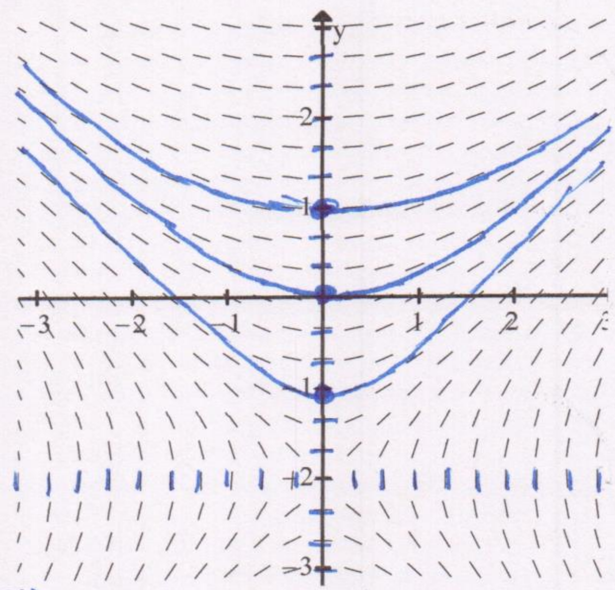
$$\int \frac{dy}{1+y^2} = \int dt$$

$$\tan^{-1}(y) = t + C$$

$$\boxed{y = \tan^{-1}(t + C)}$$

These curves do indeed look like tangent function curves!

2.



(i) Matches $y' = \dfrac{t}{2+y}$

$y' = 0$ for $t = 0$

y' is undefined for $y = -2$ (vert. tangent lines)

Solution: $\dfrac{dy}{dt} = \dfrac{t}{2+y}$

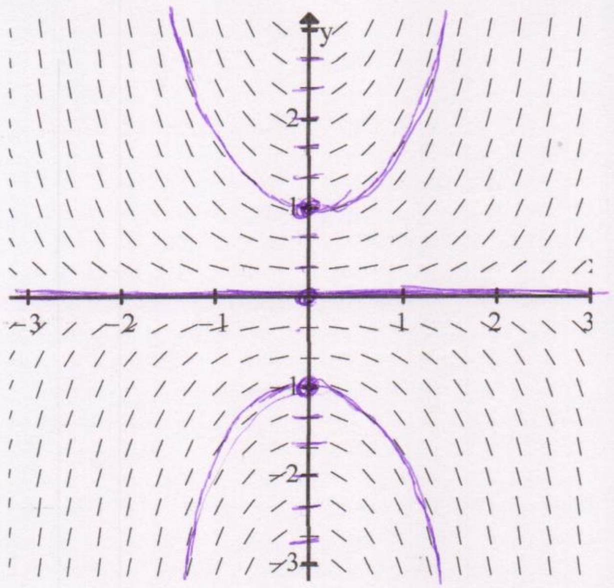
$$\int (2+y)\,dy = \int t\,dt$$

$$\boxed{2y + \tfrac{1}{2}y^2 = \tfrac{1}{2}t^2 + C}$$

Note: We could solve for y by completing the square, but we'll instead leave the solutions in IMPLICIT form

(The curves a hyperbolas, btw)

3.



(iv) Matches $y' = ty$

$y' = 0$ for $t = 0$ (y-axis)
and $y = 0$ (t-axis)

Solution: $\frac{dy}{dt} = ty$

$$\int \frac{dy}{y} = \int t\,dt$$

$$\ln|y| = \frac{1}{2}t^2 + C$$

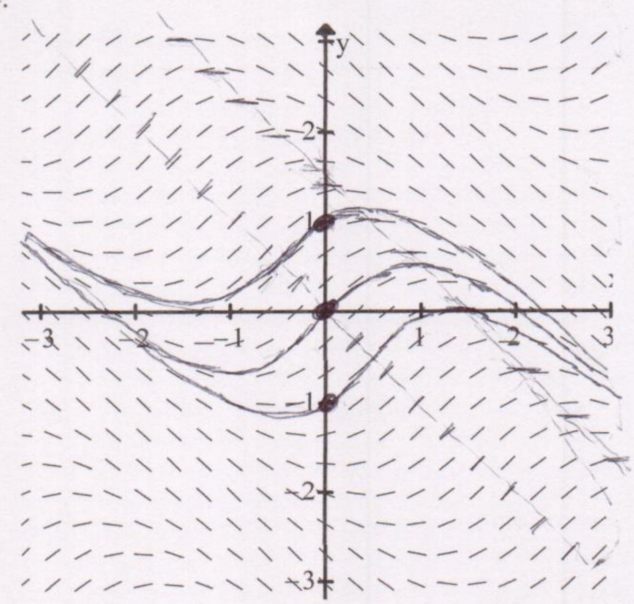
$$e^{\ln|y|} = e^{\frac{1}{2}t^2 + C}$$

$$|y| = e^C \cdot e^{\frac{1}{2}t^2}$$

$$y = \pm e^C e^{\frac{1}{2}t^2}$$

$$\boxed{y = C_1 e^{\frac{1}{2}t^2}}$$

4.



(ii) Matches $y' = \cos(t+y)$

$y' = 0$ when $\cos(t+y) = 0$

$$t + y = \frac{\pi}{2} + n\pi$$

$$y = -t + \frac{\pi}{2} \; (+n\pi)$$

1.5-ish

$y' = 1$ when $\cos(t+y) = 1$

$$t + y = 0$$

$$y = -t$$

Both look reasonable based on the graph.

Solution:

$$\frac{dy}{dt} = \cos(t+y)$$

NOT Separable!
So we can't solve this analytically, only graphically or numerically