Math 265B: Series Expansions for Select Functions

Function	Series Form	Interval of Convergence
$f(x)=e^x$	$=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots \qquad =\sum_{n=0}^{\infty}\frac{x^n}{n!}$	$-\infty<x<\infty$
$f(x)=\sin(x)$	$=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots \qquad =\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}$	$-\infty<x<\infty$
$f(x)=\cos(x)$	$=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots \qquad =\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$	$-\infty<x<\infty$
$f(x)=\ln(x)$	$=(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\frac{1}{4}(x-1)^4+\cdots \qquad =\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n}(x-1)^n$	$0<x\le 2$
$f(x)=\ln(x+1)$	$=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots \qquad =\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^n$	$-1<x\le 1$
$f(x)=\frac{1}{1+x}$	$=1-x+x^2-x^3+\cdots \qquad =\sum_{n=0}^{\infty}(-1)^n x^n$	$-1<x<1$
$f(x)=\frac{1}{1-x}$	$=1+x+x^2+x^3+\cdots \qquad =\sum_{n=0}^{\infty}x^n$	$-1<x<1$
$f(x)=\sqrt{1+x}$	$=1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3-\frac{5}{128}x^4+\cdots \qquad =\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{(1-2n)(n!)^2(4^n)}x^n$	$-1\le x\le 1$
$f(x)=\tan^{-1}(x)$	$=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots \qquad =\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1}$	$-1<x\le 1$