Math 265B: Test 1 **Name:___________________________________**

(60 points)

Credit is based on the correct work not just the final answer so please show your work in a clear, logical fashion. Simplify all answers as much as possible unless otherwise indicated and give EXACT answers unless an approximation is asked for in the problem. Only scientific calculators may be used on this exam.

1. (12 pts) Find each of the following:

(a) $\displaystyle\int \frac{t^2+5}{\sqrt{t}}\,dt$

(b) $\displaystyle\int \sec^2(x)e^{\tan(x)}\,dx$

(c) $\displaystyle\int \frac{\sin(\theta)}{\cos^3(\theta)}\,d\theta$

(d) $\displaystyle\int \frac{z+3}{z^2+6z}\,dz$

2. (5 pts) Evaluate the definite integral.

$$\int_{1}^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$$

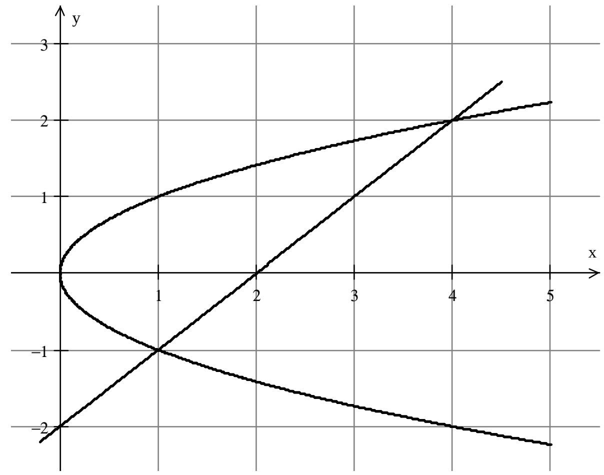
3. (9 pts) (a) Find Trap(2) to estimate the value of. $\int_{0}^{4} x^2 + 10\, dx$. Recall that the area of a trapezoid is $A = \frac{1}{2}(a+b)h$.

Sketch a graph that illustrates Trap(2) for this integral.

(b) Find the error, absolute error, and percent error for the estimation in part (a):

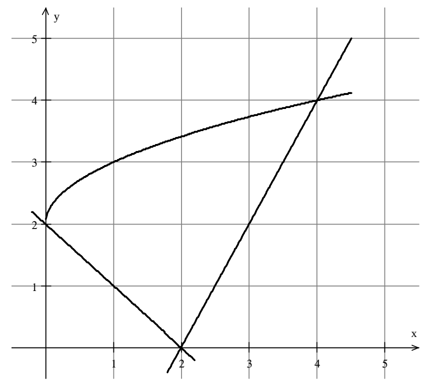
(c) If the number of subdivisions, n, were increased to be n = 20, by what factor would the error be reduced? Without attempting to find Trap(20), estimate the error for Trap(20).

4. (6 pts) Find the area between the curves $x = y^2$ and $x = y + 2$ (see graph below for reference).

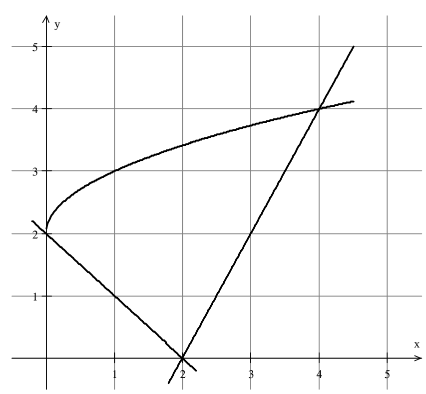


5. (6 pts) Consider the region bounded by $y = \sqrt{x} + 2$, $y = -x + 2$, and $y = 2x - 4$, as shown.

(a) Set up the integral(s) <u>with respect to x</u> that give the area of this region.
Do simplify the integrand but you do not need to evaluate the integral(s).

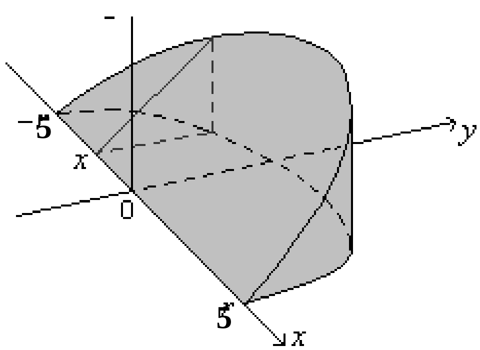


(b) Set up the integral(s) <u>with respect to y</u> that give the *area* of this region.
Do simplify the integrand but you do not need to evaluate the integral(s).

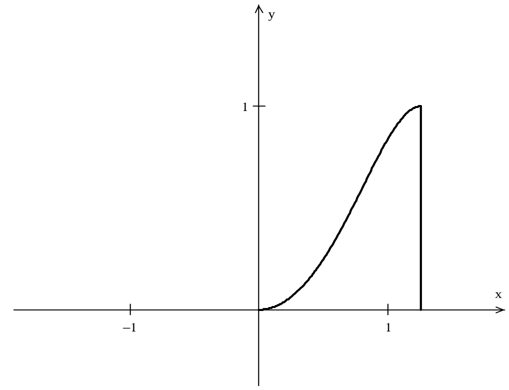


6. (6 pts) Use the Disk/Washer Method to find the volume of the solid formed by revolving the region bounded by $y = \frac{1}{2}x$, $y = x$, and $x = 2$ about the x-axis. **Include a graph of the solid in your answer**!

7. (4 pts) A wooden wedge has a semi-circular base of radius $r = 5\text{cm}$. The cross section on any plane perpendicular to the diameter of the semi-circle is a right isosceles triangle with the right angle on the semi-circle (see illustration). Calculate the volume of the wedge. (Note: The equation of the base is $y = \sqrt{25 - x^2}$.



8. (6 pts) Use the Cylindrical Shell Method to find the volume of the solid of revolution formed by revolving the region bounded by $y = \sin(x^2)$, $y = 0$ and $x = \sqrt{\frac{\pi}{2}}$ about the y-axis. (Region is shown in graph.)



Sketch the solid.

9. (6 pts) Set up the integral to find the length of the curve $y = 2e^x - \frac{1}{8}e^{-x}$ for $0 \le x \le \ln 2$
Simplify the integrand as much as possible, but you don't have to integrate.

Extra credit (4 pts): Evaluate the integral you set up above.

Math 265B: Test 1, Take Home Portion Name:_______________________________________
(40 points…8 pts per question)
Due: Tuesday, 2/17/15, at the beginning of class.

Guidelines:

- *You are welcome to work with other students in the class but the final work you hand in must be your own. Your answers must match every step of your work; otherwise, you may lose most or all of the points for the problem. Do all work by hand; i.e., no Wolfram-ing of the integrals, etc.!*
- *Please do not work with instructors, tutors, or virtual buddies on the internet.*
- *Do your work on separate paper and attach this page as a cover sheet. Make sure your work is clear, legible and well organized. Work that is poorly organized and/or difficult to read will be marked down*

1. A thin rod has density (in grams per centimeter) that varies along its length (in centimeters) according to the function $\rho(x) = \begin{cases} 3x+1 \ {}^{g}/_{cm} & 0 \le x \le 1 \ cm \\ 4-x^2 \ {}^{g}/_{cm} & 1 \le x \le 2 \ cm \end{cases}$

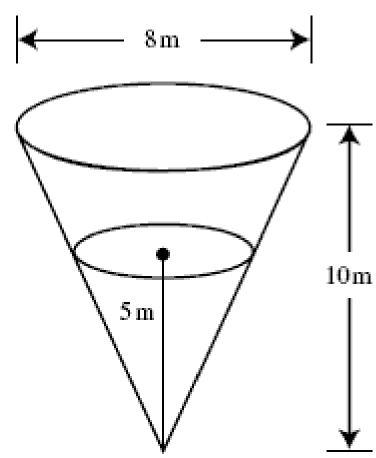
(a) Use calculus to determine the total mass of the rod.
(b) Extra credit (4 pts): Research how to find the Center of Mass then find this point on the rod. Show work!

2. Cars are designed with a “crumple zone" in the front of the car. The more “crushed” the front becomes in the event of a crash, the more it will resist further crushing, so we can model the forces and energy involved in the same way we do a linear spring*.

In a safety test, it was found that a for certain type of car, a force of $2.1\times10^6\,\text{N}$ would compress the front of the car 0.2 meters.
(a) Assuming the crumple zone obeys Hooke’s Law, find the spring consant, k, for this type of car.
(b) If the same type of car is in an accident where the the front of the car crumples (compresses) 0.5 meter, use calculus to determine the amount of energy the crumple zone absorbed.

3. A tank in the shape of a cone with the flat portion on top (*as shown)* has a radius of 4 meters and a height of 10 m.. The tank is half-filled with gasoline which has a density of $\rho = 737\,\frac{kg}{m^3}$. Use calculus to determine the amount of work that is done against gravity to pump all of the gasoline to a point 10 meters above the tank. Express the answer in two ways:

(1) In exact form, in terms of ρ, π, and g
(2) In approximate form, with three significant figures.



4. Derive the arc length integral, used for finding the length of a curve, $y = f(x)$ on the interval [a,b]. Clearly explain every step of the derivation (in words). Be sure to include a Riemann Sum in your work and show/explain how that sum transforms into an integral.

5. (a) Set up the integral (using either Shells or Disks/Washers) to find the volume of the solid formed by revolving the region contained in the boundaries $y = e^x + e^{-x}$, $x = 0$ and $y = \frac{5}{2}$ about the line $x = 3$. Sketch the solid. (Hint: Use Wolfram Alpha to help you solve the equation $\frac{5}{2} = e^x + e^{-x}$, but write out the steps!).

(b) Use Wolfram Alpha (or any other Computer Algebra System) to evaluate the integral.