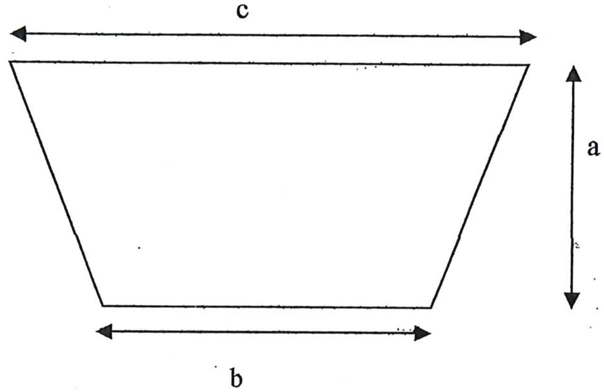
Math 265B: Test 1 Take Home Name:______________________________

_________/15 points

- *You may work with other students in the class but the final work you hand in must be your own. Your answers must match every step of your work; otherwise, you may lose most or all of the points for the problem.*
- *Please do not ask instructors, tutors or anyone else to solve these problems for you!*
- *Do your work on separate paper and attach this page as a cover sheet. Make sure your work is clear, legible and well organized. Work that is poorly organized and/or difficult to read will be marked down.*
- *The test is due at the beginning of class on Tuesday 2/18/20.*

1. Find the total force that the water in a reservoir exerts on a trapezoidal dam with the following dimensions:
   Height = a meters
   Length of base = b meters
   Length of top edge = c meters

   Assume the dam is full. Express your answer in terms of a, b and c.



2. A series of circular gates that allow water to spill from the dam are positioned in the lower half of the dam (see picture).

Determine the force of the water on the cover of one of these circular gates if the top of the gate is located 100 feet below the top of the dam (again assume the dam is full).
The diameter of the gate is 4 meters.

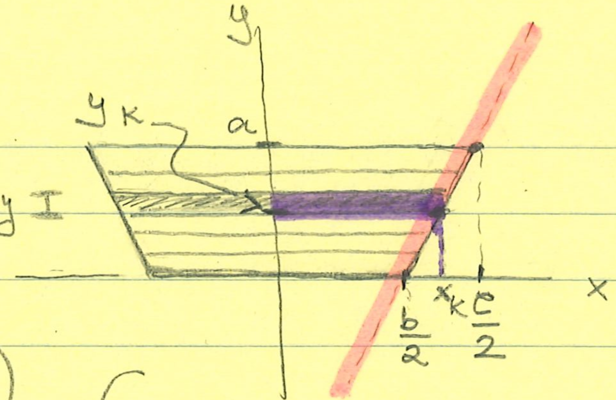
100 feet

$\approx 100 \text{ ft} \cdot \frac{1 \text{ m}}{3.28 \text{ ft}}$

≈ 30.5 m

(note: answers will slightly vary depending on conversion factor and rounding

Note: This is the picture of a gate that was being worked on.



1. Force = Pressure · Area
= ρg · Depth · Area

$$F \approx \sum_{K=1}^{n} \rho g \begin{pmatrix}\text{Depth}\\ \text{of slice}\end{pmatrix}\begin{pmatrix}\text{Area}\\ \text{of slice}\end{pmatrix}$$

$$= \sum_{K=1}^{n} \rho g (a - y_K)\, \cancel{2} \boxed{\left[\frac{(c-b)}{\cancel{2}a} y_K + \frac{b}{\cancel{2}}\right]} \Delta y$$

Depth $= a - y_K$

Area = width · Δy
$= 2x_K \cdot \Delta y$

$\Downarrow$

To find x_K:

(1) Find line equation:

$$m = \frac{\Delta y}{\Delta x} = \frac{a - 0}{\frac{c}{2} - \frac{b}{2}} = \frac{a}{\frac{c-b}{2}}$$

$$m = \frac{2a}{c-b}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{2a}{c-b}\left(x - \frac{b}{2}\right)$$

$$y = \frac{2a}{c-b}\left(x - \frac{b}{2}\right)$$

(2) Solve for x:

$$\frac{(c-b)y}{2a} = x - \frac{b}{2}$$

$$\boxed{x = \frac{(c-b)}{2a} y + \frac{b}{2}}$$

Taking the limit as $\Delta y \to 0$ $(n \to \infty)$ converts the sum to an integral

$$\boxed{F = \int_0^a \rho g (a-y)\left(\frac{c-b}{a} y + b\right) dy}$$

Now we need to evaluate this integral

(1) Simplify integrand:

$$(a-y)\left(\frac{c-b}{a} y + b\right)$$

$$= (c-b)y + ab - \frac{(c-b)}{a} y^2 - by$$

$$= -\frac{(c-b)}{a} y^2 + (c-2b)y + ab$$

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so $F = \int_0^a \rho g \left\{ (a-y)\left(\frac{(c-b)}{a} y + b\right) \right\} dy$

* becomes $= \rho g \int_0^a \left\{ -\frac{(c-b)}{a} y^2 + (c-2b) y + ab \right\} dy$

(2) integrate $= \rho g \left[-\frac{(c-b)}{3a} y^3 + \frac{(c-2b)}{2} y^2 + aby \right|_0^a$

(3) evaluate $= \rho g \left[\frac{-(c-b)a^3}{3a} + \frac{(c-2b)}{2} a^2 + ab \cdot a - (0+0+0) \right]$

Since there are some like terms floating around in this mix, we should clean up and simplify AMAP (as much as possible).

(4) simplify

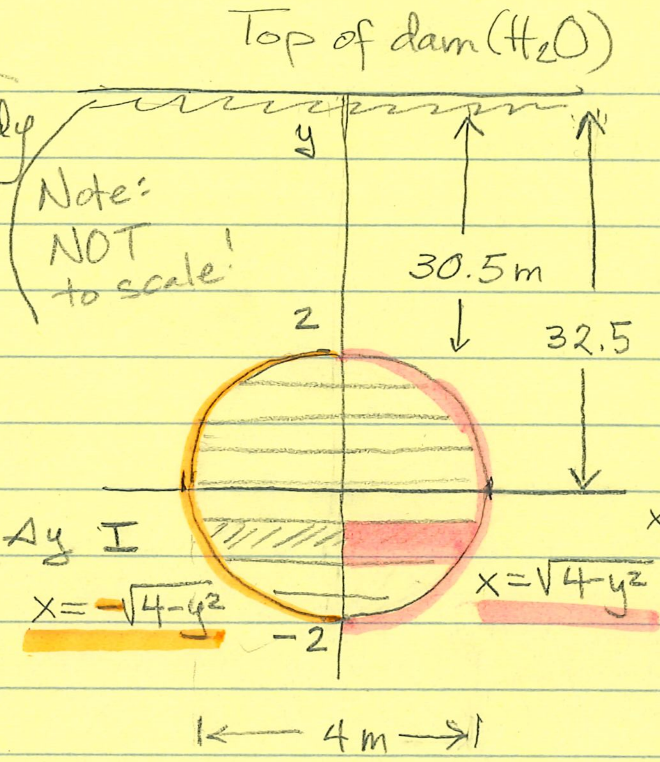
$$-\frac{(c-b)}{3} a^2 + \frac{a^2 c}{2} \underbrace{\cancel{- a^2 b + a^2 b}}_{\to 0}$$

$$= \frac{2(b-c)}{6} a^2 + \frac{3a^2 c}{6} = \frac{1}{6}\left[2a^2 b - 2a^2 c + 3a^2 c\right]$$

so $F = \rho g \left[\frac{2a^2 b + a^2 c}{6} \right]$

Note: Result from Wolfram: $\rho g \cdot \frac{1}{6} a^2 (2b+c)$
(without rho and g)

2. $F = \displaystyle\int_{-2}^{2} \rho g \overbrace{(32.5-y)}^{\text{depth}} \cdot \overbrace{2\sqrt{4-y^2}\,dy}^{\text{Area}}$



$= 2\rho g \displaystyle\underbrace{\int_{-2}^{2} (32.5-y)\sqrt{4-y^2}\,dy}_{\text{Wolfram}}$

$\approx 2\rho g\,(204.204)$

$= 2(1000)(9.8)(204.204)$

$\approx 4 \times 10^6$ N ← Best answer

(4,002,984 N)

too precise { Given measurements are no way near this level of accuracy!!

Depth $= 30.5 + 2 - y$

$= 32.5 - y$

Width $= 2x$

$x^2 + y^2 = 4$

$x = \pm\sqrt{4-y^2}$

Width $= 2\sqrt{4-y^2}$