Math 265B: Test 2 (Spring 2020-Wright) **Name:** KEY

For maximum credit, please show all of your work in a clear, logical manner. Simplify all answers as much as possible.

In-class: ______________/80 points Take Home: ________________/20 points

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Helpful Identities: $\sin^2\phi = \frac{1}{2}(1-\cos 2\phi)$ $\cos^2\phi = \frac{1}{2}(1+\cos 2\phi)$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

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1. (9 pts) Given the integral $\int \frac{1}{x^2-4}\,dx$

(a) Could you use Partial Fraction Decomposition to integrate? $\boxed{\text{YES}}$ NO

If yes, what would the decomposition be? $\rightarrow$ $\frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)} = \boxed{\frac{A}{x-2} + \frac{B}{x+2}}$

Just the form
don't solve for A & B

(b) Could you use Trig Substitution? $\boxed{\text{YES}}$ NO

If yes, what would the substitution be? $\boxed{x = 2\sec\theta}$

$dx = 2\sec\theta\tan\theta$

(c) Could you use u-Substitution (Reverse the Chain Rule/Guess-and-Check)? YES $\boxed{\text{NO}}$

If yes, what would u be? No, there is no "du" available in the integrand

If $u = x^2 - 4$ then $du = 2x$

$\Rightarrow$ need an x in the integrand

2. (2 pts) Suppose you made the Trig Substitution of $x = 2\sin\theta$ in an integral.

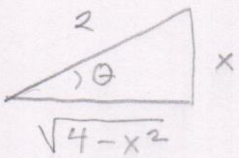
Which of the following would be true? Show work for credit!

$\frac{x}{2} = \sin\theta$

a) $\cos\theta = \frac{\sqrt{2-x^2}}{x}$

$\boxed{\text{b) } \tan\theta = \frac{x}{\sqrt{4-x^2}}}$

Work:



c) both of these

d) neither of these

LIATE

3. (9 pts) Use Integration by Parts to integrate:

$\int x^2 e^{3x}\, dx$.

$u = x^2 \qquad dv = e^{3x}$

$du = 2x \qquad v = \frac{1}{3}e^{3x}$

$u = x \qquad dv = e^{3x}$

$du = 1 \qquad v = \frac{1}{3}e^{3x}$

$= \frac{1}{3}x^2 e^{3x} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}\, dx\right]$

$\boxed{= \frac{1}{3}x^2 e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C}$

easiest!

Column Method

u		dv
x^2	$(+)$	e^{3x}
$2x$	$(-)$	$\frac{1}{3}e^{3x}$
2	$(+)$	$\frac{1}{9}e^{3x}$
0		$\frac{1}{27}e^{3x}$

Note: $\int e^{kx}\, dx$

$\int x^2 e^{3x}$

$= x^2\left(\frac{1}{3}e^{3x}\right) - 2x\left(\frac{1}{9}e^{3x}\right) + 2\left(\frac{1}{27}e^{3x}\right) + C$

$\boxed{= \frac{1}{3}x^2 e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C}$

OR $\frac{1}{3}e^{3x}\left(x^2 - \frac{2}{3}x + \frac{2}{9}\right) + C$

4. (10 pts) Use Partial Fraction Decomposition to integrate:

$\int \frac{4x^2+3}{x^2(x^2+1)}\, dx. = \int \frac{\cancel{A}^{\to 0}}{x} + \frac{B}{x^2} + \frac{\cancel{Cx} + D}{x^2+1}\, dx$

$= \int \frac{3}{x^2} + \frac{1}{x^2+1}\, dx$

$\frac{4x^2+3}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$

$= \int 3x^{-2} + \frac{1}{x^2+1}\, dx$

$4x^2 + 3 = Ax(x^2+1) + B(x^2+1) + x^2(Cx+D)$

$x = 0: \boxed{3 = B}$

5. (8 pts) Use Identities to integrate:

$\int \sin^2(\theta)\cos^3(\theta)\, d\theta$.

odd power on $\cos\theta$ $\rightarrow$ peel off $\cos\theta$

$$= \int \sin^2\theta \, \underline{\cos^2\theta} \cdot \overbrace{\cos\theta \, d\theta}^{du}$$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

$$= \int \sin^2\theta \, (1 - \sin^2\theta) \cos\theta \, d\theta$$

$$= \int u^2 (1 - u^2) \, du$$

$$= \int u^2 - u^4 \, du$$

$$\rightarrow \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$$

$$\boxed{= \frac{1}{3}\sin^3\theta - \frac{1}{5}\sin^5\theta + C}$$

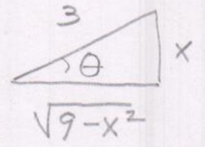
6. (8 pts) Use Trig Substitution to evaluate the definite integral: $\int \frac{1}{(9-x^2)^{3/2}} dx$.

$x = 3\sin\theta$

$dx = 3\cos\theta$

$\frac{x}{3} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$



$$\int \frac{1}{(9-x^2)^{3/2}} dx$$

$$= \int \frac{1}{(9 - 9\sin^2\theta)^{3/2}} \cdot 3\cos\theta \, d\theta$$

$$= \int \frac{3\cos\theta \, d\theta}{[9(1-\sin^2\theta)]^{3/2}}$$

[note: $9^{3/2} = 27$]

$$= \int \frac{\cancel{3}^{\,1}\cos\theta}{\cancel{27}_{\,9}\,(\cos^2\theta)^{3/2}} d\theta$$

$$= \frac{1}{9}\int \frac{\cancel{\cos\theta}^{\,1}}{\cos^{\cancel{3}_{\,2}}\theta} d\theta$$

$$\rightarrow = \frac{1}{9}\int \frac{1}{\cos^2\theta} d\theta$$

$$= \frac{1}{9}\int \sec^2\theta \, d\theta$$

$$= \frac{1}{9}\tan\theta + C$$

$$\boxed{= \frac{1}{9}\frac{x}{\sqrt{9-x^2}} + C}$$

even powers $\Rightarrow \frac{1}{2}$ angle ID

7. (8 pts) Use Identities to integrate: $\int \sin^2(t)\cos^2(t)dt$

$$\int \sin^2(t)\cos^2(t)\,dt$$

$$= \int \frac{1}{8} - \frac{1}{8}\cos(4t)\,dt$$

$$\boxed{= \frac{1}{8}t - \frac{1}{32}\sin(4t) + C}$$

$$\sin^2(t)\cdot\cos^2(t)$$

$$= \frac{1-\cos(2t)}{2}\cdot\frac{1+\cos(2t)}{2}$$

$$= \frac{1-\cos^2(2t)}{4} = \frac{\sin^2(2t)}{4}$$

Better

$$= \frac{1}{8}(1-\cos(4t))$$

$$= \frac{1-\frac{1}{2}(1+\cos(4t))}{4}$$

$$= \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos(4t)$$

$$\frac{1}{8}$$

8. (4 pts) How do you check (by hand!) that an integration was done correctly? differentiate!

Check whether the integration below is correct. Show work for credit and do not check by integrating!

$$\int \frac{8x^2+24}{x^3+4x}\,dx = \ln\left(x^2+4\right)+6\ln(x)+C$$

Correct!

$$\frac{d}{dx}\left[\ln(x^2+4)+6\ln(x)\right]$$

$$= \frac{2x}{x^2+4} + \frac{6}{x}$$

$$= \frac{2x(x)+6(x^2+4)}{x(x^2+4)} = \frac{8x^2+24}{x^3+4x} \checkmark$$

9. (6 pts) Evaluate the definite integral. Show work for credit. Leave your answer exact (in terms of "ln").

$$\int_1^2 \ln x\,dx$$

$u = \ln x \quad dv = 1$

$du = \frac{1}{x} \quad v = x$

$$= x\ln x - \int_1^2 x\left(\frac{1}{x}\right)dx$$

$$= x\ln x - \int_1^2 1\,dx = x\ln x - x\Big|_1^2$$

$$= \left[2\ln 2 - 2\right] - \left[1\cancel{\ln(1)}^{\,0} - 1\right]$$

$$\boxed{= 2\ln 2 - 1}$$

Typo I didn't catch! $n-1$ not $n=1$

10. (16 pts) A formula for Trap(n) is $Trap(n) = \sum_{k=0}^{n=1} \frac{f(x_k)+f(x_{k+1})}{2} \Delta x$

(a) Show how this formula can be rewritten as $Trap(n) = \left[\frac{1}{2} f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \frac{1}{2} f(x_n)\right] \Delta x$

Do this by expanding the first few terms of the first summation formula.

$$\sum_{k=0}^{n-1} \frac{f(x_k)+f(x_{k+1})}{2} = \frac{f(x_0)+f(x_1)}{2} + \frac{f(x_1)+f(x_2)}{2} + \frac{f(x_2)+f(x_3)}{2} + \cdots$$

there will be a matching term $+ \frac{f(x_{n-1})+f(x_n)}{2}$

$$= \frac{1}{2} f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \frac{1}{2} f(x_n)$$

$$= \frac{1}{2} f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \frac{1}{2} f(x_n) \quad \blacksquare$$

(b) Find Trap(2) to estimate the value of. $\int_0^4 16 - x^2 \, dx$. Illustrates Trap(2) on the given graph

$n = 2$

$\Delta x = \frac{4-0}{2} = 2$

$x_0 = 0$

$x_1 = 2$

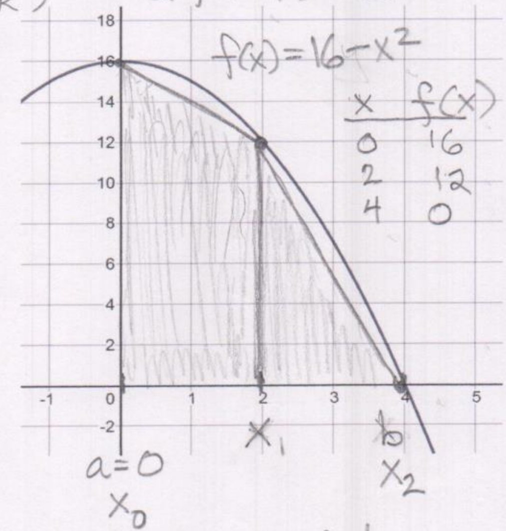
$x_2 = 4$

$$Trap(2) = \left[\frac{f(0)+f(2)}{2} + \frac{f(2)+f(4)}{2}\right] \cdot \Delta x$$

(1st formula)

$$= \left[\frac{16+12}{2} + \frac{12+0}{2}\right] \cdot 2$$

$$= 20 \cdot 2 = \boxed{40 \text{ unit}^2}$$



(c) Find the error and absolute error of your estimation in (b).

Error = Est − Actual

$= 40 - \int_0^4 16 - x^2 \, dx$

$= 40 - \frac{128}{3}$

$= -\frac{8}{3} = -2.\overline{6}$

$\int_0^4 16 - x^2 dx$

$= 16x - \frac{1}{3}x^3 \Big|_0^4$

$= 64 - \frac{64}{3}$

$= \frac{128}{3}$

underestimate!

Error: $-2.\overline{6} \approx -2.667$

Absolute Error: $+2.\overline{6}$

(d) If the number of subdivisions, n, were increased to be n = 20, by what factor would the error be reduced?

$n = 2$ to $n = 20 \Rightarrow k = 10$

Answer: $10^2 = 100$

(e) Without attempting to find Trap(20), estimate the error for Trap(20), using your work from the previous parts of this problem.

Answer: $\frac{-2.667}{100} = -.02667$