**Math 265B: Test 2 Take Home** (30 pts) Name: KEY

Due: Thursday, 2/27/14, at the beginning of class.

***Guidelines:***

- *You are welcome to work with other students in the class but the final work you hand in must be your own. Your answers must match every step of your work; otherwise, you may lose most or all of the points for the problem. Do all work by hand; i.e., no Wolfram-ing of the integrals, etc.!*
- *Please do not work with other instructors, tutors or virtual buddies on the internet.*
- *Do your work on separate paper and attach this page as a cover sheet. Make sure your work is clear, legible and well organized. Work that is poorly organized and/or difficult to read will be marked down*
- *The solution for #2 and #3 should include* ***large, clear, well-labeled graphs****. The graphs should clearly show the coordinate system you're using in the problem and all of the values for the set up should pertain to that coordinate system.*

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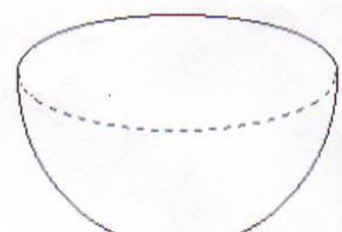
1. Cars are designed with a "crumple zone" in the front of the car. The more "crushed" the front becomes in the event of a crash, the more it will resist further crushing, so we can model the forces and energy involved in the same way we do a linear spring*.

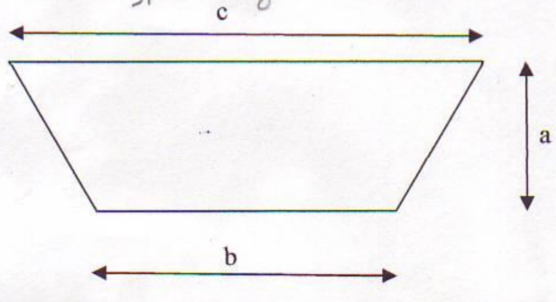
   Suppose that for a certain car, an impact of $1.35\times10^{6}\,\text{N}$ will compress the front of the car 0.5 meters. Use calculus to determine the amount of energy absorbed if the front of the car crumples (compresses) 0.8 meters.

2. A tank in the shape of a hemisphere with the flat portion on top (*as shown*) has a radius of 4.00 meters. The tank is half filled with liquid mercury which has a density of 13534 kg/m$^3$ Use calculus to determine the amount of work that is done against gravity to pump all of the mercury to a point 3 meters above the tank. Express the answer in two ways:



   (1) In exact form, in terms of $\rho$ and $g$ and $\pi$
   (2) In approximate form, with three significant figures.

3. Use calculus to find the force that the water in a reservoir exerts on a trapezoidal dam with the following dimensions: "a" feet is the height, "b" feet is the length of the base, set against the ground, and" c is the length of the top edge, which is the top of the dam. Assume the dam is full. Express your answer in terms of a, b and c, $\rho$ and $g$




* Admittedly this is an oversimplification of the real world situation but modeling can begin with simplified assumptions then later add more "reality", hence more complexity, to refine the model.

1.) Linear spring: $F = kx$ $\quad F = 1.35 \times 10^6\ N$

$x = 0.5\ m$

$$\Rightarrow k = \frac{1.35 \times 10^6\ N}{0.5\ m}$$

$$k = 2.7 \times 10^6\ \frac{N}{m}$$

"Crushing" $\Rightarrow$ compression

We've been choosing compression as the negative direction, but the net result will be the same if we choose compression to be the positive direction.

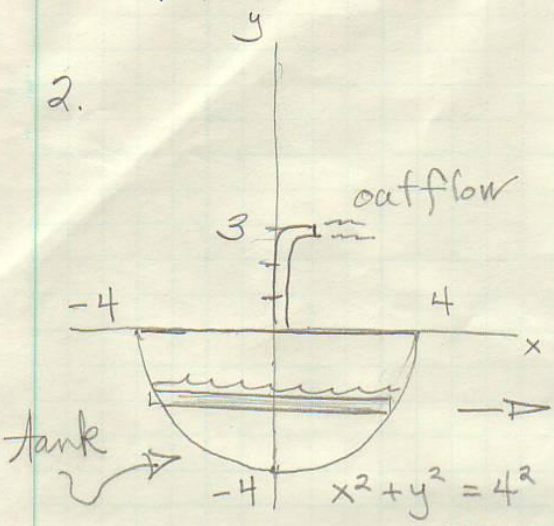
$$\text{Energy} = \int_0^{0.8} 2.7 \times 10^6\ x\ dx$$

$$= 2.7 \times 10^6 \left[ \tfrac{1}{2} x^2 \right|_0^{.8}$$

$$= 8.64 \times 10^5\ N\text{-}m\ (\text{or } J)$$

Had we set the integral up with

2.




$\rho_{Hg} = 13{,}534 \ \frac{kg}{m^3}$

$r = 4m$

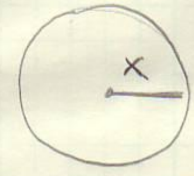
$d(y) = 3 - y$ = distance slice must move

$x^2 + y^2 = 4^2$

$x = \pm\sqrt{16 - y^2}$

(Right half: $x = \sqrt{16 - y^2}$

→ cross section:




$A(y) = \pi r^2$

$= \pi x^2$

$= \pi\left(\sqrt{16 - y^2}\right)^2$

$= \pi(16 - y^2)$

$$\text{Work} = \int_{\text{bottom slice}}^{\text{top slice}} \rho g A(y) d(y) \, dy$$

$$= \int_{-4}^{-2} \rho g \left(\pi(16 - y^2)\right)(3 - y) \, dy$$

$$= \rho g \pi \int_{-4}^{-2} 48 - 3y^2 - 16y + y^3 \, dy$$

$$= \rho g \pi \left[48y - y^3 - 8y^2 + \tfrac{1}{4}y^4\right|_{-4}^{-2}$$

$$= \rho g \pi \left(\left[48(-2) - (-2)^3 - 8(-2)^2 + \tfrac{1}{4}(-2)^4\right] - \left[48(-4) - (-4)^3 - 8(-4)^2 + \tfrac{1}{4}(-4)^4\right]\right)$$

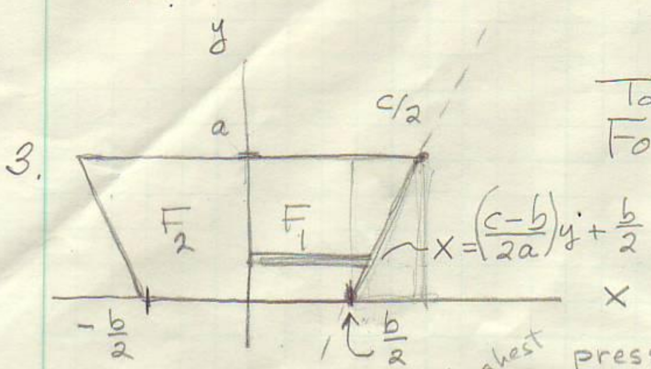
$$= \rho g \pi (76)$$

part 1: Work $= 76 \rho g \pi$ Joules

part 2: Work $\cong$ 31,700,000

$= 3.17 \times 10^7$ Joules

3.




Total Force $= F_1 + F_2$

$= 2F_1$ by symmetry

$$2F_1 = 2\int_{\substack{\text{lowest}\\ \text{slice}}}^{\substack{\text{highest}\\ \text{slice}}} \underbrace{\rho g \,\underset{\text{of slice}}{\text{depth}}}_{\text{pressure}} \cdot \underbrace{\underset{\text{of slice}}{\text{Width}}\, dy}_{\text{area}}$$

$\rho_{H_2O} = 1000 \frac{Kg}{m^3}$

line equation

$m = \dfrac{a}{\frac{c}{2} - \frac{b}{2}} = \dfrac{2a}{c-b}$

$y - 0 = \frac{2a}{c-b}\left(x - \frac{b}{2}\right)$

$\left(\frac{c-b}{2a}\right)y = x - \frac{b}{2}$

$x = \left(\frac{c-b}{2a}\right)y + \frac{b}{2}$

(This is the width of the slice in the $F_1$ region.)

$$= 2\int_0^a \rho g (a-y)\cdot\left(\frac{c-b}{2a}y + \frac{b}{2}\right)dy$$

$$= \cancel{2}\rho g \int_0^a \frac{c-b}{\cancel{2}}y + \frac{ab}{\cancel{2}} - \frac{c-b}{\cancel{2}a}y^2 - \frac{b}{\cancel{2}}y \, dy$$

$$= \rho g\left[\frac{(c-b)}{2}y^2 + aby - \frac{1}{3}\frac{(c-b)}{a}y^3 - \frac{b}{2}y^2\right]_0^a$$

$$= \rho g\left[\frac{(c-b)a^2}{2} - \frac{(c-b)a^2}{3} - \frac{a^2b}{2} + a^2b\right]$$

$$= \rho g\left[\frac{(c-b)a^2}{6} + \frac{3}{3}\cdot\frac{a^2b}{2}\right]$$

$$= \frac{\rho g a^2}{6}\left[c + 2b\right]$$

So the total force on the dam is

$$F = \frac{\rho g a^2}{6}\left[2b^* + c\right] \quad *$$

(units would be Newtons)

$*$ Reasonability check. Since b is at the bottom of the dam where the pressure is greatest, its size should affect F more than c, which it does.