Math 265B: Test 3 Name:____

For maximum credit, please show all of your work in a clear, logical manner. Simplify all answers as much as possible. Leave answers in exact form unless an approximation is asked for in the instructions.

1. (3 pts) True or false:

(a) Euler's Method gives the exact solution to an initial value problem. TRUE FALSE

(b) The parameterization of a curve is always unique;
i.e., there is only one set of parametric equations for any curve. TRUE FALSE

(c) The polar coordinates $(2,0)$ and $(-2,\pi)$ describe the same point. TRUE FALSE

2. (6 pts) CHECK (don't attempt to solve!) to see whether $y = t + \frac{1}{t}$ is a solution to the differential equation $ty' + y = 2t$. Be sure to state your conclusion (the function is or is not a solution to the DE). *Show work that justifies your answer*.

3. (4 pts) Which of the following differential equations are separable? (Circle those that are.)

(i) $y' = t^2 y^2$ (ii) $y' = 3t + y$ (iii) $y' = (y-1)(y+3)$ (iv) $y' = \sin(ty)$

4. (8 pts) Find the general solution (in explicit form) for each differential equation. Carefully show all steps when simplifying your answer.

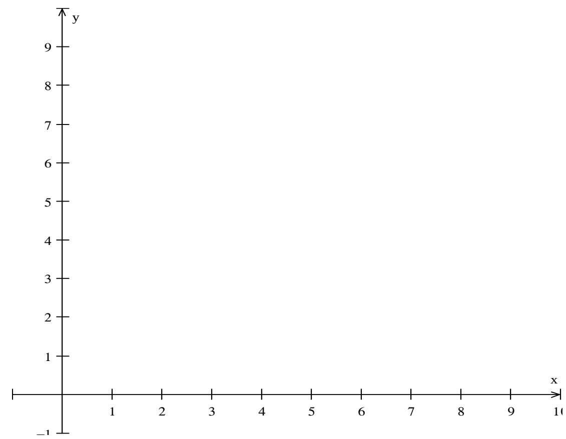
(a) $y' = \dfrac{3+t^2}{\sqrt{y}}$ (b) $y' + 5y = 10$

5. (8 pts) Solve the Initial Value Problem.

$$\frac{dy}{dt} = e^{-y}\sin(t), \quad y(\tfrac{\pi}{2}) = 0$$

6. (8 pts) Consider the Initial Value Problem $\dfrac{dy}{dx} = (y-3)(y-6), \quad y(0) = y_0$.

(a) Sketch the slope field for the differential equation.



(b) Find any equilibrium (constant) solutions and classify each as stable or unstable.

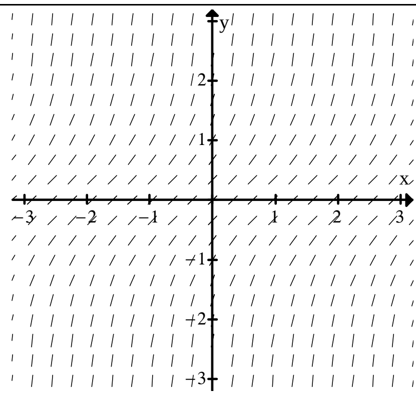
(c) Sketch solution curves for the Initial Conditions $y(0) = 1$ and $y(0) = 7$

(d) For what initial conditions (i.e., what values of y_0) will the solution curves be decreasing functions?

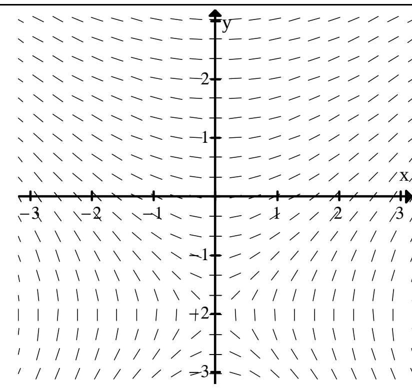
7. (8 pts) Label each slope field with the differential equation that best matches it. No work is required to be shown.

(i) $y'(t) = \dfrac{t}{2+y}$ (ii) $y'(t) = \cos(t+y)$ (iii) $y'(t) = 1 + y^2$ (iv) $y'(t) = ty$

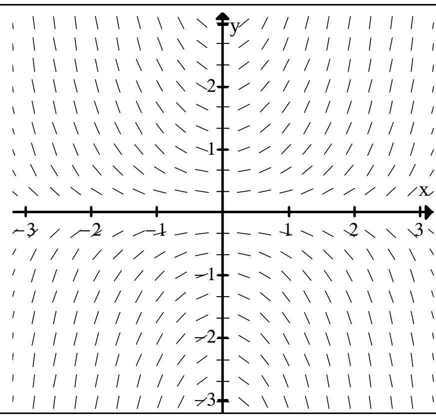
(a)



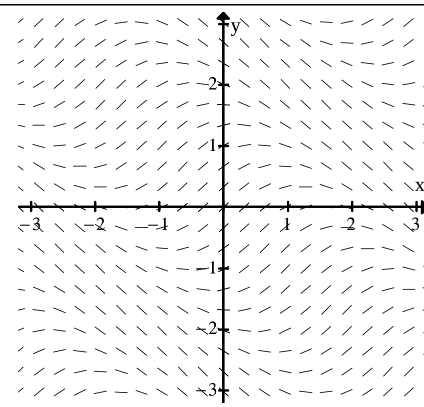
(b)



(c)



(d)

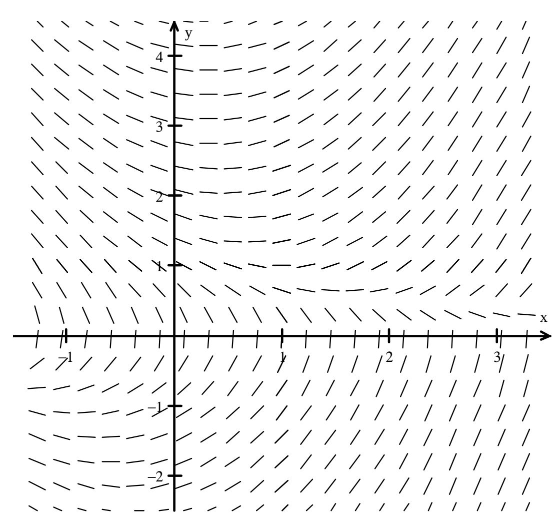


8. (10 pts) Consider the I.V.P. $y' = t - \frac{1}{y}$ $y(0) = 2$

(a) Use Euler's Method, with $\Delta t = 1$ to estimate the value of $y(2)$. Clearly show all of your work.

$y(2) \cong$ ___________________

(b) On the slope field provided, sketch the polygonal Euler's solution "curve", based on your work from part (a).



9. (4 pts) A cup of coffee cools at a rate proportional to the difference between the coffee's temperature, H, and the ambient temperature, A.

Set up a differential equation that models this relationship. Do not solve the D.E.

10. (12 pts) Consider the curve described by the parameterization $\begin{cases} x = 3 - t^2 \\ y = 1 - t \end{cases} \quad 0 \le t \le 3$

(a) Graph the curve. Include the orientation and label the t-values for each point in your graph.

(b) Find the *parametric form* of the tangent line equation at the point where t = 1. Sketch the line on your graph from part (a).

(c) Eliminate the parameter and express the curve as an xy-equation.

11. (12 pts) Find a parameterization of each of the following curves. Be sure to include the domain!

(a) The segment of the parabola $y = x^2 + 3$ where $-1 \le x \le 3$

(b) The line segment that begins at point P(4, -5) and ends and point Q(1,-1)

(c) A circle of radius 3, oriented clockwise, with center (3, 2) and period of 10 seconds.

(d) The top half of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, oriented counter clockwise, starting at point (2, 0)

12. (6 pts) (a) Plot the point with polar coordinates $\left(4, \frac{\pi}{3}\right)$

(b) Find two other sets of polar coordinates that locate that same point. One set must have a negative r-value.

(c) Convert the point into rectangular coordinates. For full credit, express your answer in EXACT terms.

13. (6 pts) Convert each of the xy-equations into a polar equation.

(a) $x^2 + y^2 = 7$

(b) $y = -6$

(c) $y = x$ (express your final answer in terms of θ only).

14.(4 pts) Graph the region $\left\{(r,\theta) \middle|\ 2 \le r \le 3,\ -\frac{\pi}{6} \le \theta \le \frac{\pi}{6}\right\}$