Math 265B: Test 4 - In Class Portion Name:___________________________________

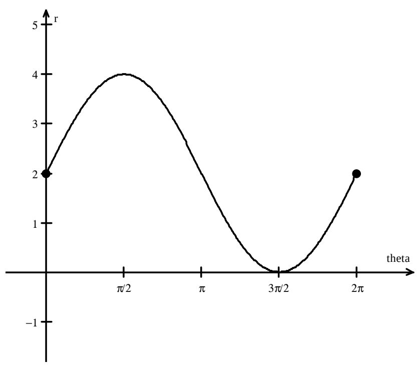
(76 pts)

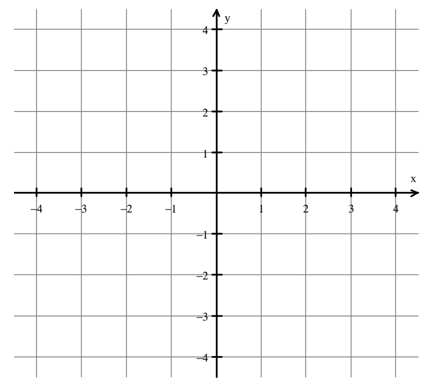
For maximum credit, please show all of your work in a clear, logical manner. Simplify all answers as much as possible. Leave answers in exact form unless an approximation is asked for in the instructions.

Helpful formula: $\dfrac{dy}{dx} = \dfrac{r' \sin\theta + r\cos\theta}{r' \cos\theta - r\sin\theta}$

1.(4 pts) Use the Cartesian r, θ graph to graph the polar equation in the xy-coordinate system.

$r = 2 + 2\sin\theta$

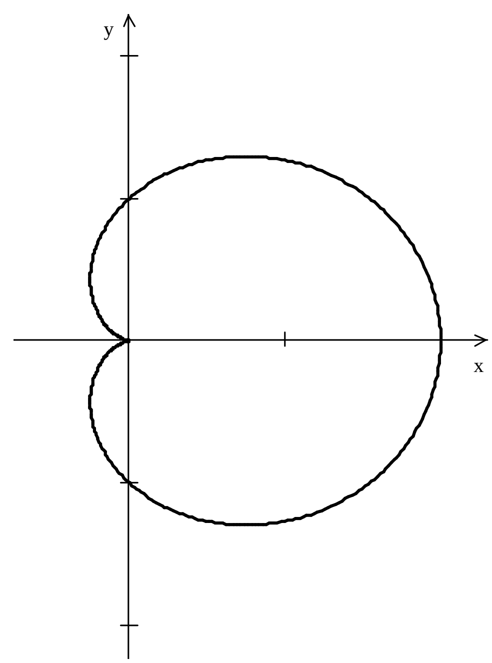




2. (6 pts) Consider the polar curve $r = 1 + \cos\theta$.

Use calculus to determine the slope of the curve at the point $\left(\frac{3}{2}, \frac{\pi}{3}\right)$.

Plot the point on the curve, sketch the tangent line, and label it with the slope.

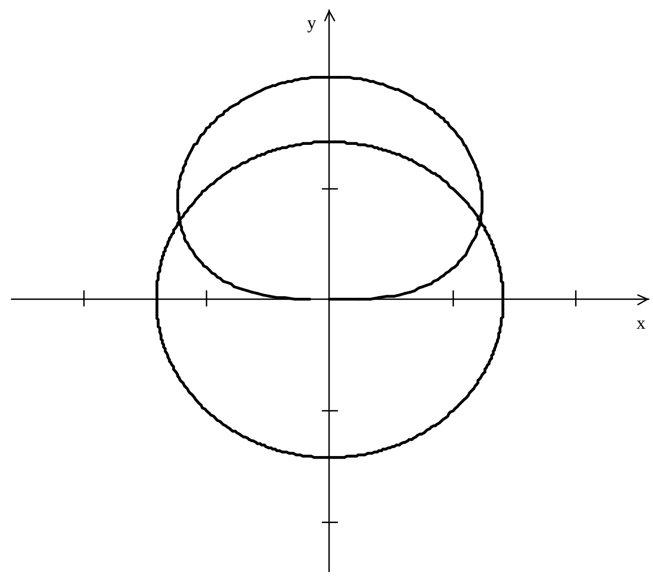


Extra credit (4 pts) (do this only if you've finished the entire exam!):

Use the graph to estimate the values of θ at which the curve has a vertical tangent line. Verify your guess analytically. i.e., algebraically solve for your original guess. (Do this calculation on the next page.)

Extra credit work (optional):

3. (8 pts) (a) Find the points of intersection for the polar curves $r = \frac{1}{\sqrt{2}}$ and $r = \sqrt{\sin\theta}$. The graph of the curves is given.



Find the area of the region, that is <u>outside</u> of the curve $r = \frac{1}{\sqrt{2}}$ and <u>inside</u> the curve $r = \sqrt{\sin\theta}$.

4. (14 pts) Using analytic techniques, i.e. *formally*, find the following limits. The methods/theorems you may use include the Ranking Theorem, L'Hopital's Rule, algebraic manipulation, and/or the Squeeze Theorem.
If the limit does not exist, briefly explain why. Caution: THESE ARE NOT SERIES! DO NOT USE SERIES CONVERGENCE TESTS!

(a) $\lim\limits_{n\to\infty} \frac{n^{50}}{2^n}$

(b) $\lim\limits_{n\to\infty} \frac{n^2 - n}{4n^2 + 1}$

(c) $\lim\limits_{n\to\infty} \frac{\tan^{-1}(n)}{n}$

(d) $\lim\limits_{n\to\infty} \cos(\pi n)$

For rest of the exam, you may find any limits "by inspection".

5. (8 pts) Given the series $\sum_{k=0}^{\infty} \frac{3}{k+1} - \frac{3}{k+2}$

(a) Find the first 3 partial sums of the series.

(b) What is the *nth* partial sum of the series?

(c) Use your work above to show that the series converges and to find the sum of the series. Show careful work that justifies your answer.

6. (6 pts) Write each of the following series in $\sum$ notation, and determine whether the series converges or diverges. You do not have to prove convergence/divergence, just determine "by inspection".

(a) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots =$ ______________________ converges diverges

(b) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots =$ ______________________ converges diverges

(c) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} \cdots =$ ______________________ converges diverges

7. (6 pts) If the series below converges, find its sum. Show work. If it doesn't converge, say how you can tell.

$$\sum_{k=0}^{\infty} 2\left(\tfrac{3}{4}\right)^k + 4\left(-\tfrac{1}{3}\right)^k$$

For each of the following problems, 8 - 11, determine convergence and organize your work in the "Claim…Work…Conclusion" format.

8. (5 pts) Consider the series $\sum_{k=0}^{\infty} \frac{1}{k^2+1}$. Use Direct Comparison to <u>prove</u> that the series converges or diverges.

9. (5 pts) Use the Ratio Test to <u>prove</u> that the series converges or diverges.

$$\sum_{k=1}^{\infty} \frac{k^3}{3^k}$$

10. (8 pts) Use any test to prove convergence or divergence of the series.

(a) $\sum_{k=1}^{\infty} (-1)^k \frac{k^3+1}{k^3+2k}$

(b) $\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!}$

11. (6 pts) Determine whether the series is *absolutely convergent*, *conditionally convergent*, or *divergent*.

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k^{2/5}}$$

Is the series absolutely convergent?______________

Show work to justify your answer:

Is the series conditionally convergent?______________

Show work to justify your answer:

Math 265B: Test 4, Take Home Portion Name:____
(24 points…3 pts per question)
Due: Thursday, 5/14/15, at the beginning of class.

Guidelines:

- *You are welcome to work with other students in the class but the final work you hand in must be your own. Your answers must match every step of your work; otherwise, you may lose most or all of the points for the problem.*
- *Please do not work with instructors, tutors, or virtual buddies on the internet.*
- *Do your work on separate paper and attach this page as a cover sheet. Make sure your work is clear, legible and well organized, with the proper notation. Work that is poorly organized and/or difficult to read will be marked down*

Note: For all questions involving Taylor series or polynomials, you don't have to derive the polynomial/series unless the question asks you to.

1. Determine the interval of convergence for the following series "by inspection". Explain your reasoning.

$$\sum_{k=0}^{\infty}\left(\frac{x-1}{3}\right)^k$$

2. Use the <u>Ratio Test Method</u> to determine the <u>radius of convergence</u> and <u>interval of convergence</u> for the given series.

(a) $\sum_{k=1}^{\infty}\frac{(-2)^k}{k}x^k$ (b) $\sum_{k=1}^{\infty}\frac{k^2}{k!}x^k$

3. (a) <u>Derive</u> the first two non-zero terms of the Taylor series about x = 0 for the function $f(x)=e^{x^2}$

(b) Find the Taylor series again by using the Taylor series formula for e^x (you don't have to derive the e^x series).

4. <u>Derive</u> the 3rd degree Taylor polynomial for $\sqrt{1+x}$ about x = 0 then use it to approximate $\sqrt{0.96}$. Assuming your calculator gives the exact value, determine the absolute error of the estimation (give your answer with 3 significant figures).

5. Use the Taylor series (at x = 0) for $\ln(1+x)$ to find a series for $x^2\ln(1+4x)$.
Write the first 4 terms of the series then write the series using $\sum$ notation with all terms inside the $\sum$
Determine the interval of convergence for the new series.

6. Find $\lim_{x\to 0}\frac{e^x-e^{-x}}{x}$ using series:

7. Use the first 3 terms of the series for the integrand to estimate the value of the integral: $\int_1^2 \frac{\sin(x^2)}{x^2}dx$

8. Use series to derive Euler's Formula: $e^{i\theta}=\cos\theta+i\sin\theta$