Math 265B: Review for Test 4

The following topics would be good to review for the exam. There are some suggested review problems at the end of the review sheet.

Remember to include the Polar Coordinate graphs worksheet with your homework.

Calculus in Polar Coordinates (11.2, 11.3)

- Use the graph of $r = f(\theta)$ in the (r,θ) plane to graph in the (x, y) plane (see the worksheet on polar graphs for an example)

- Find the slope of the tangent to a polar curve. Find points at which a polar curve has a vertical tangent and at which it has a horizontal tangent. The formula for $\frac{dy}{dx}$ will be given.
- Set up an integral to find the area of region determined by polar coordinates.

Sequences:

- Use analytic (formal) methods to find the limit of a sequence. You will have to be able to apply techniques such as algebraic manipulation, the Squeeze Theorem, and L'Hopital's Rule.

- Be able to identify and explain why the limit of a sequence doesn't exist; specifically, note when the terms oscillate with no damping, or oscillate without bound and thus don't approach a single value as n increases.

- Know the Ranking Theorem and apply it in finding limits.

Note: There are two approaches to finding limits. One is to find the limit intuitively "by inspection", using the concept of dominance for example. This is a loose way to find limits but is very useful.

The other approach is to find the limit formally using analytic methods such as those mentioned above. On the exam, I will ask you to find limits "formally" in some problems. Later, when you're applying the Tests for Convergence, you may (and should be able to!) find limits by inspection.

Infinite Series: (these are series of constants)

- Given an expanded series, be able to find the kth term of the series, and put it into $\sum a_k$ notation.

- Be able to find partial sums, S_n, of a given infinite series. Use the partial sum to determine convergence/divergence of the series. Examples: Telescoping Series, Alternating Series.

- Use Properties of Series to find the sum of a series.

- Be able to determine and PROVE the convergence or divergence of a series using the following tests:

    - Divergence Test
    - Geometric Series Test…be able to find the sum as well if the series converges
    - Integral Test
    - p-Test
    - Direct Comparison
    - Limit Comparison
    - Ratio Test
    - Alternating Series Test

- Be able to explain, intuitively, what each of the tests above tells you about the behavior of the series and why the series does or does not converge based on that behavior.

- Be able to determine whether an alternating series converges absolutely, converges conditionally, or diverges.

- Know that the "tails" of series are what determine divergence or convergence of a series

Chapter 10: This material will be covered on the Take Home portion of the exam

Taylor Polynomials and Taylor Series:

- Given a function *f*, find the Taylor Polynomial of a given degree, expanded about either $x = 0$ and $x = a$. Use the polynomial for estimation.

- Be able to find the Taylor series for a new function by manipulating a known series. The manipulations include substitution, differentiation, and integration of a known series.

Power Series: (these are series that contain a variable…looks like a polynomial that has infinitely many terms)

Given a power series , determine (a) the radius of convergence and (b) the interval of convergence

Suggested Review Problems:
Chapter 11 Review, page 775: 29, 33, 37 + be sure to look over the polar graphing worksheet!

Chapter 9 Review, page 679: 1, 3 , 6, 8, 9, 11, 13, 14, 20, 23, 25, 29, 33, 37, 43, 44, 48 (For #23 – 48, focus on determining what test would be effective to use…suggestions are given in the answers below).

Important notes on select problems and even answers:
Chapter 9:
#1d: : On 1d, this is just semantics. If a series converges absolutely then we wouldn't also say it converges conditionally because saying the latter implies it ONLY converges conditionally.

An analogy would be contrasting acceptance to Cal Poly either "conditionally" or "absolutely". Absolute acceptance would mean regardless of this semester's outcome, you're in. Conditional acceptance means you have to successfully complete this semester's load of classes. If you had absolute acceptance, then passed all your classes, and then someone asked you what kind of acceptance you had, it would be incorrect to say you were conditionally accepted even though you passed the classes. You already had the stronger form of acceptance.

#3: Be able to say why the limit is 0; limit is 0 by the Ranking Theorem.
#6: limit is 0, multiply by $\frac{n+\sqrt{n^2+1}}{n+\sqrt{n^2+1}}$ then evaluate the limit.
#8: limit dne due to oscillation with no damping thus terms do not approach a single number as n increases; contrast that with $a_n = \frac{\sin(n)}{n}$, which oscillates but is damped by the 1/n factor and converges by the Squeeze Theorem (Be able to show this!).

#9: limit dne due to oscillations that increase without bound. This is a geometric series with r > 1.
#14: Sum is 5/6
#20: Break into 2 series and evaluate each individually. Sum is - ½
Suggested Tests:
#23 (p-Test), #25 (Geometric Series Test), #29 (Direct comparison Test), #37 (Ratio Test), #44 (Diverges by Divergence Test), #48 (Converges conditionally…use Integral Test to show that it does NOT converge absolutely)